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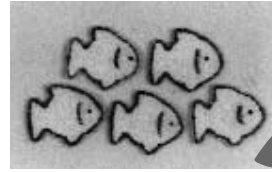
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Path Games

- ▲ Hurry! You need to roll at least a three to get past the cave.
- ▲ Let's say we get to move ahead one space every time we land on a gold star.
- ▲ If you give me two of your jewels, I can have three jewels on each crown.
- ▲ You don't have to count all of the dots. See, you just say "three" and then count these: 4, 5, 6, 7, 8.

Children often make these types of comments as they play teacher-developed path games. Their excitement, thinking, and peer interactions are evident. In order to resolve problems presented by the games, children often use mathematical strategies that are more complex than would be required by workbooks or many commercial math games. They get immediate feedback when they try to prove a point or when a peer disagrees. Since children are encouraged to be autonomous, they can alter the rules of the games as long as all the players agree. They can also increase the difficulty of the games or play the games in new ways that are individually appropriate for them.

Teachers' Questions

What are path games?

In path games, children roll dice to advance a mover along a path made of clearly defined spaces. Path games encourage social interaction as children engage in mathematical thinking and problem solving. Teachers can design the games to coordinate with topics of interest, field trips, or books in the classroom.

Why is it important to include path games in the math curriculum?

Path games incorporate the thinking strategies needed for grid games (see chapter 4) at a more difficult level and place additional emphasis on social interactions with teachers and peers. Moving

along a path is more abstract than taking counters; therefore, path games are cognitively more difficult than grid games. Path games provide opportunities for children to discuss their ideas with others and be confronted with different opinions. When children defend their beliefs, they may strengthen their knowledge base or be forced to rethink how they view a situation. For example, when one child moves one space for each dot shown on the dice while the other child moves randomly, or when one child re-counts the space his mover is on and the other child does not, one or both of them may be bothered by the results. Over a period of time these interactions help children move toward more logical thinking.

What makes a good path game?

A good path game has a clearly defined path and intriguing movers that encourage children to move in one-to-one correspondence. It involves a topic that is interesting to children and often coordinates with other aspects of the curriculum. Good path games are attractive and durable.

What is wrong with commercial path games?

Commercial games generally do not encourage children to construct mathematical relationships or use their own thinking strategies. They are often too difficult or too easy. Some games, such as Chutes and Ladders, incorporate a long (100-space) and confusing path with a relatively simple 1-6 spinner. Games such as Candyland encourage children to use color matching rather than quantification strategies. (It is likely that these games were never intended to be mathematical materials.) Teachers may be unable to coordinate commercial games with other curriculum areas. Also, commercial math games do not provide opportunities for teachers to assess or plan for individual children.

What are the rules for path games?

There are no specific rules for path games. Children are encouraged to decide among themselves what the rules will be so that they can adapt the games to correspond to their own developmental levels. Children who are not being evaluated prefer more challenging material (see "Your Praise Can Smother Learning" by David L. Martin in *Learning*, Feb. 1977, p. 44). Therefore teachers need not worry that children will consistently play games below their cognitive level. In fact, children often create more difficult problems than teachers would ever think to

give them. For example, a four-year-old child decided to double every roll of the dice and consistently did this throughout a game.

Children usually want to roll dice in order to tell how many spaces to move when playing path games. This seems to be a standard convention.

By kindergarten most children elect to play games competitively. They enjoy playing to win. This seems to be a natural part of development and occurs even in classrooms where teachers de-emphasize competition. As children get older, the excitement and fun of competition motivates them to play path games again and again.

What concepts emerge when children play path games?

As children interact with each other while playing path games, they continue to consider one-to-one correspondence, equivalence of sets, and addition, but at a more complex level.

Path games require children to think about one-to-one correspondence in a new context. Rather than take one counter for each dot on the dice, they must move one space along the path for each dot. This is more abstract because children cannot see a specific quantity of items to correspond to the number of dots on the dice.

When children construct equivalent sets in path games, the sets they must compare (the number of dots on the dice versus the number of spaces moved) are not as concrete as when they roll dice and take counters. They have to construct the idea that they can quantify spaces on the path before they can consider comparing a set of spaces to a number rolled on a die. This is why children can successfully play grid games before they are able to play path games, and why short path games are introduced before long path games.

Long path games are an ideal format for encouraging the addition of two dice, since the games have many spaces and children are eager to get to the end! Kindergarten children enjoy path games so much that many learn all the addition combinations just by playing the games.

What kinds of path games can teachers make and what affects their level of difficulty?

Teachers can make short path, long path, continuous path, and collection games. The level of difficulty of path games is affected by whether there are separate paths for each player or just one path for all players and by the length of the path. The number of dots on the die (1-3 versus 1-6), the number of dice used, and

the addition of traps or collection spaces also affects the difficulty. Since the types of games vary in difficulty, teachers can select a specific type of path game to best approximate the cognitive level of specific children in the class.

Short path games are designed to help children transition from grid games to longer path games. Two identical paths are provided so that two children can parallel play. The two paths may be opposite each other on one game board or on separate game boards for younger children who have trouble sharing a single board. The paths should be straight and clearly delineated to avoid the perceptual confusion that curves sometimes cause. The paths generally consist of 10 to 12 spaces each, since more than twelve spaces can be overwhelming, while fewer spaces look like a grid rather than a path. A 1-3 die or spinner is used to determine the number of spaces to move, since a 1-6 die may be too challenging for children in this transition stage. In addition, a 1-6 die could end the game in only two rolls!

Long path games are used when children are ready for more challenging games. They contain 25 to 50 spaces along a curved path and may involve a variety of traps and bonus spaces that require the player to roll again, go back one space, or follow more original directions that the children themselves design. One or two 1-6 dice are used since children who can play long path games can usually quantify to six and may be ready to start adding two dice together.

Continuous path games do not have a definite start and finish point. The path has 25 to 50 spaces and is usually shaped in a square, oval, or circle. One or two 1-6 dice are used.

Collection games are variations of short path, long path, and continuous path games. Collection pieces on short path games encourage children to play them for a longer period of time. When collection pieces are used on a short path game, a shorter path may be desirable. Long path collection games may include special spaces where children stop and collect counters, or children may collect counters at the end of the path. Players may compare each other's quantities of items they have collected, or they may record their quantities on a piece of paper or on a graph. Long or continuous path collection games can provide an additional challenge for kindergarten children.

What errors do children make when playing path games?

Children make counting errors such as skipping over objects or double-counting as well as addition errors. Counting errors often continue for an extended period. However, when children make errors they are often corrected by their peers. Children who are challenged in their thinking by other children are forced to rethink their logic and ultimately move ahead in their thinking and stop making errors.

Another common error that children make is to re-count the space their mover occupies when they begin a new turn. Some children stop making this error after playing gross-motor path games (see activity 7.3) where they themselves are the mover.

Children who successfully use grid games often need a period of exploration with path games before they are able to play them in a conventional manner. Initially, some children roll the dice and hop along the path without regard to the quantity on the dice. They do not yet perceive the relationship between the roll of the dice and the spaces on the path. For some children the total length of a long path may be overwhelming. They may respond by moving from start to finish quickly.

What is the teacher's role?

The teacher plans developmentally appropriate games, encourages social interaction, and stimulates higher-level thinking strategies through questioning techniques or appropriate modeling.

Teachers must provide a variety of levels of games in order to best meet the developmental needs of an entire class. Grid games, short path, and long path games may all be appropriate for certain children. Teachers might want to create games on these three levels that all center around a common topic, such as autumn. Then, any child in the class who wanted to play an autumn game could find one on her level (activities 5.7, 5.12, and 5.18). This does not mean that the teacher should suggest that a child not play a particular game. Rather, the teacher should allow the children to play the game *in their own way* and perhaps afterward suggest another game that might be more appropriate.

Path games, especially long path games, require more social interaction among the participants than grid games, which are sometimes used by children in parallel-play fashion. The teacher should observe and facilitate these interactions but not offer solutions. The teacher maintains the role of a mediator to help children see each other's viewpoints and further their thinking. Younger children often seek the teacher as a partner for playing path games. They seem to enjoy a close relationship with the

teacher. Most kindergarten children prefer peer interactions, but they may need the teacher to help mediate disputes or assist in problem solving.

The teacher can model alternative thinking strategies when it is her turn to play. For example, if a child has been adding two dice together by counting all the dots for an extended period, the teacher can model “adding on.” After rolling a five and a three, the teacher, touching the five die, then touching each dot on the three die, might say, “I rolled a five; six, seven, eight.” When the teacher models, she should not appear to indicate that the child’s approach is incorrect. The teacher can model some social conventions of game playing for younger children, such as using the dice to determine how far to move, turn taking, and the meaning of traps and bonus spaces.

What materials are needed for path games?

Path games require poster board, stickers or rubber stamps, dice, and individual movers. These are essentially the same materials described for grid games. Unlike grid games, where the counters for each child are similar, path games should provide identifiable movers for each player. If the movers are identical, children cannot remember which mover is theirs. Children are drawn to path games because of the interesting board and movers. Look for movers such as the following:

- ▲ commercial plastic or wooden people, especially those that exhibit cultural diversity
- ▲ erasers
- ▲ party favors
- ▲ magnets
- ▲ wooden or resin figures
- ▲ cake decorations

What design criteria should teachers consider?

Teachers must carefully plan the path design, choose the movers, and select the type and number of dice. They should give attention to the aesthetic qualities of the game and consider the following issues:

- ▲ The topic should be interesting to children so they are motivated to play the game. It might coordinate with another aspect of the curriculum such as a book, field trip, or a thematic topic.

- ▲ The length and configuration of the path, including any traps or bonus spaces, should be appropriate for the developmental level of the children. In general, older preschool children and kindergarten children need longer paths.
- ▲ The dice should involve quantities appropriate for both the length of the path and the developmental levels of the children. Young preschoolers typically use one 1-3 die, while older preschool and kindergarten children may use one to two standard 1-6 dice.
- ▲ The path should be clear and not confusing.
- ▲ The game should be attractive and durable.
- ▲ The movers should correlate with the topic of the game.
- ▲ Illustrations and print should be neat and relevant to the game.

What pitfalls should teachers avoid?

Path games will be most successful if teachers consider the following:

- ▲ Spaces should not touch each other. This makes it difficult for children to count as they move.
- ▲ The movers should not cover more than one space at a time. Otherwise the children will be confused as they attempt to quantify the spaces of the path.
- ▲ Avoid movers with wheels. Children tend to roll them along the path instead of quantifying one space at a time.
- ▲ Teachers should measure the area where the game will be used to be certain that the game board will fit.
- ▲ The decorative detail should not make the board cluttered or confusing.

How should path games be displayed?

A math game table is an ideal place for children to use path games. A “math center” can stimulate children’s interest in math games, similar to the way “writing centers” increase their attention to written language.

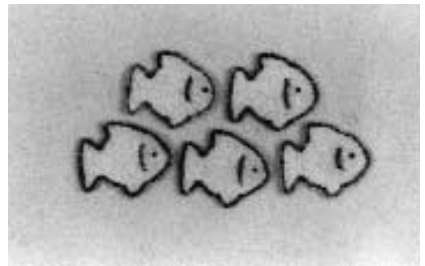
Children can play short path games on the floor or on a low bench, but long path games are too large to fit easily into the

manipulative area. Children become frustrated if others step on game boards or knock over movers. Therefore, a game table is preferable for long path games. Children and parents often use the game table as a transitional activity upon arrival or at departure.

How can teachers use path games as assessment tools?

Path games provide teachers with further opportunities to assess children through observation. Teachers can use the same assessment form for path games that they use to record anecdotal data from math manipulatives and grid games. (See the assessment form examples in chapter 2 and the appendix, section A.) They can use the “comments” section to record additional information such as “re-counts space already occupied.”

Path Game Activities



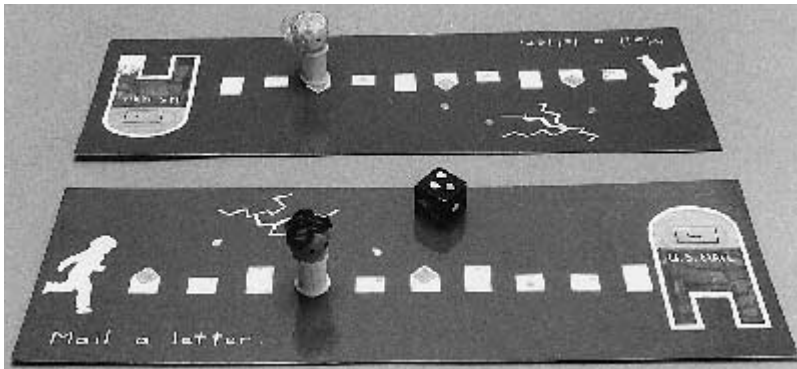
5.6 Letter Short Path Game

Book

This game is designed to coordinate with *A Letter to Amy* by Ezra Jack Keats (Harper Row, 1968), in which a little boy writes a letter to invite a friend to his birthday party. On the way to the mailbox, he drops the letter as the wind blows during a rainstorm.

Materials

- ▲ 2 pieces of poster board, each 6 by 18 inches
- ▲ 20 envelope and letter stickers to form two separate paths, each with 10 spaces
- ▲ silhouette stickers for the start points
- ▲ mailbox illustrations for the end points
- ▲ two wooden or plastic people to use as movers
- ▲ 1-3 die or spinner



Child's Level

This game is most appropriate for children who can quantify to three and are just beginning to play short path games. The path is straight, short, and very clear.

What to Look For

Children may advance along the path according to the roll of the die.

Some children may hop along the path to the mailbox without regard to the amount shown on the die.

Helpful Hint

Use white correction fluid to create the lightning.

Modifications

Add more people movers to encourage children to move along the path several times.

Change to a 1-6 die if a 1-3 die is too easy.

Provide small pieces of paper at the end of the path for children to write letters or their names.

Questions to Extend Thinking

How many more spaces do you have to move to mail your letter?

If I move two more, will I be just as close to the mailbox as you are?
What do you need to roll to reach the mailbox?

Integrated Curriculum Activities

Include the books *The Jolly Postman* by Janet and Allan Ahlberg (Little, Brown, 1986) and *A Letter to Amy* in the reading area.

Share rain and wind songs with the children.

Include the letter grid game (activity 4.10) in the manipulative area.

Add the letter long path game (activity 5.16), if appropriate for your group.

Set up a pulley in the gross-motor area so children can mail letters (activity 7.12).

5.12 Autumn Continuous Path Collection Game

Materials

- ▲ poster board, 15 by 15 inches
- ▲ 24 self-adhesive circles ($\frac{3}{4}$ inch) to form a continuous path
- ▲ 5 basket stickers for collection spaces
- ▲ basket of 25 to 50 acorns for collection pieces
- ▲ 2 small squirrel movers, each a different color
- ▲ 1-6 die



Child's Level

This game is most appropriate for children who can quantify to six and have had experiences with short path games and larger

quantities of counters.

What to Look For

Children may move along the path according to the roll of the die and collect an acorn when the squirrel lands on a basket sticker.

Some children may play with the acorns.

Some children will count or sort the acorns.

Some children will hop the squirrels around the path without regard to the die.

Some children may place one acorn on each circle or place several acorns on each basket.

Helpful Hint

Use a glue gun to secure loose acorn caps to the nuts.

Modifications

Use two dice for children who are ready for addition.

Provide other types of nuts or seed pods for variety.

Kindergarten children may want to graph how many of each type they collect.

Questions to Extend Thinking

What should happen when a squirrel lands on a basket?

How many more spaces do you need to move to reach a basket?

What happens if both squirrels land on the same space?

Integrated Curriculum Activities

Include autumn books such as *Nuts to You!* by Lois Ehlert (Harcourt Brace Jovanovich, 1993) in the reading area.

Create a nut collection (activity 3.15).

Select another autumn path game (activities 5.7 or 5.18) that is appropriate for your group.

Plan a nut tasting activity.

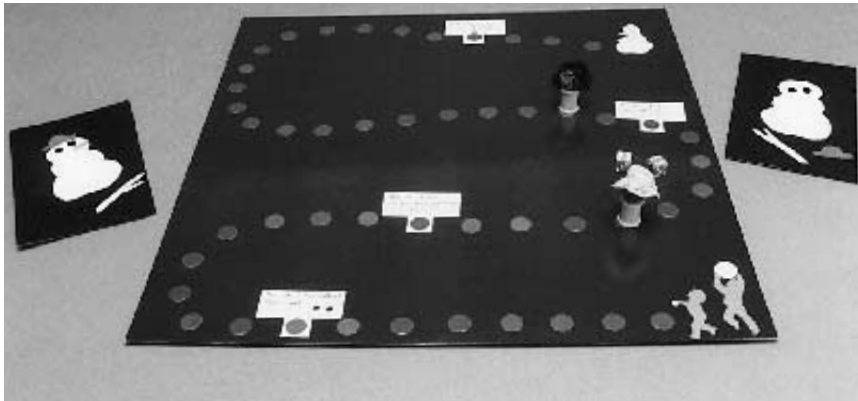
Take a nature walk.

Put large nuts, tongs, and buckets in the sensory table.

5.2/ Winter Long Path Game

Materials

- ▲ poster board, 22 by 22 inches
- ▲ 49 self-adhesive circles ($\frac{3}{4}$ inch) to form the path
- ▲ construction paper and stickers (carrot, scarf, hat, and coal) for the bonus spaces
- ▲ snowman sticker at the end of the game
- ▲ 2 small felt boards, 6 by 8 inches
- ▲ felt pieces (2 each of a snowman, hat, carrot, and scarf; multiple pieces of coal)
- ▲ small wooden or plastic people to use as movers
- ▲ 2 standard 1-6 dice



Child's Level

This game is most appropriate for children who have played complex path games and enjoy trap/bonus spaces. The bonus spaces require some knowledge of addition combinations. Older preschool and kindergarten children find this game challenging.

Helpful Hint

Be prepared for younger children to ignore the path game and simply play with the felt board pieces for an extended period. After that, a well-timed question may draw their attention to the game.

Children who cannot yet read will remember the bonus directions because they are so unique..

What to Look For

Children may roll one or two dice to determine how far to move along the path.

Some children will advance along the path but ignore the bonus space directions, for example, "roll an 8 to collect the scarf."

Some children will advance along the path with the intent of landing on the bonus spaces. They may change direction just to return to a bonus space they missed, or they may choose not to move until they finally roll the number they need to land on a bonus space.

Some children will add two dice together by counting all the dots.

Some children will ignore the dice and play with the flannelboard pieces.

Modifications

Due to the complexity of this game, no modifications are recommended. Give ample time for children to explore possibilities.

Questions to Extend Thinking

What do you need to roll to reach the first bonus space?

How many ways can you roll a twelve to collect the hat?

What happens if you advance past the bonus space without stopping?

How do you get your carrot if you miss the carrot space?

Integrated Curriculum Activities

Include winter books such as *The Snowy Day* by Ezra Jack Keats (Viking Press, 1962), *The Mitten* adapted by Jan Brett (G. P. Putnam's Sons, 1989), and *The Jacket I Wear in the Snow* by Shirley Neitzel (Greenwillow, 1989) in the reading area.

Share winter songs and poems with the children.

Create a snowflake collection (activity 3.16).

Include the snowman grid game (activity 4.8) or a snowflake grid game (activity 4.11 or 4.16) in the manipulative area.

Include the winter short path game (activity 5.10) for children not yet ready for long path games.

Estimate how many snowmen are in a clear plastic jar.

4.14 Palm Tree Grid

Predictable Book

This game was designed to correlate with *Chicka Chicka Boom Boom* by Bill Martin, Jr., and John Archambault (Simon & Schuster, 1989), a predictable book where alphabet letters climb a coconut tree.



Materials

- ▲ 2 grids, 12 by 4 inches, each with 6 palm tree stickers
- ▲ 2 sets of small alphabet letters
- ▲ 1-4 die
- ▲ 2 baskets or coconut half-shells to display alphabet letters

Child's Level

Although this grid has only six spaces, it is intended for more advanced children. The large quantity of counters (26 letters) is overwhelming for less experienced children.

What to Look For

Children may roll the die and place a corresponding number of letters onto a palm tree sticker.

Some children may use the alphabet to spell words or simply talk about letters.

Modifications

Provide more alphabet letters.
Use a 1-6 die.

Helpful Hints

Small alphabet erasers make inexpensive counters.

Young children may have trouble using letters as counters. They may be able to think of letters only in relation to their names.

Questions to Extend Thinking

How can you divide the alphabet among the six palm trees?

Do you have just as many letters as I do?

Do you have enough letters to put three onto each palm tree?

Are there enough for four per tree?

Integrated Curriculum Activities

Include the book *Chicka Chicka Boom Boom* in the reading area.

Include the palm tree game (activity 2.9) in the manipulative area.

Use coconut shells as tone blocks.

Use coconut shells as scoops in sand or water.

Put a variety of coconuts in the science area.